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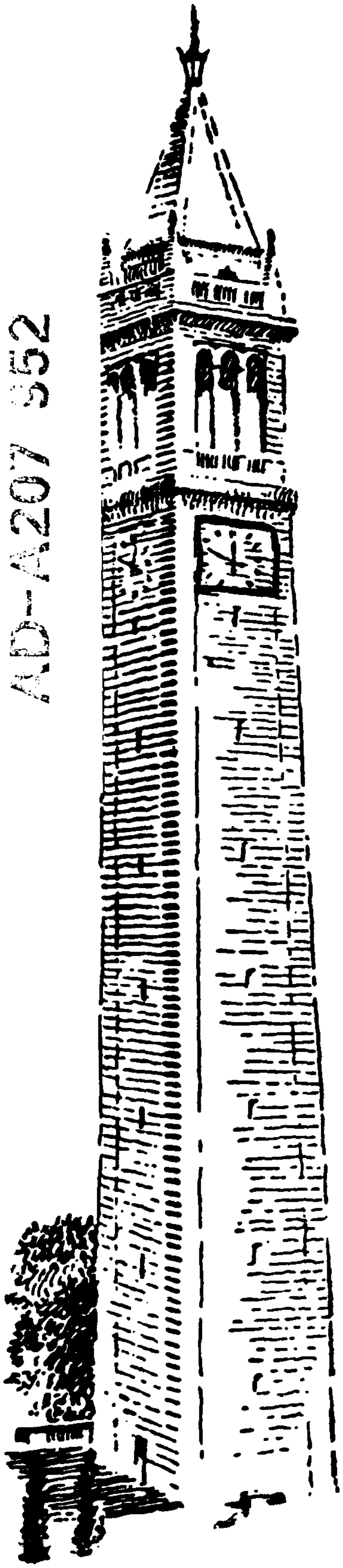

FIRST & SECOND QUARTER PROGRESS REPORT 1988 ON PLASMA THEORY AND SIMULATION

January 1 to June 31, 1988

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SAIC Subcontract No. 11-880132-61,
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ELECTRONICS RESEARCH LABORATORY
College of Engineering
University of California, Berkeley, CA 94720

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Our group uses theory and simulation as tools in order to increase the understanding of plasma instabilities, heating, transport, plasma-wall interactions, and large potentials in plasmas. We also work on the improvement of simulation both theoretically and practically.

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Research in plasma theory and simulation, plasma-wall interactions, large potentials in plasmas.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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20. ABSTRACT

General Plasma Theory and Simulation

A. A magnetized plasma next to an absorbing wall is simulated, showing positive wall charging causing a large E-field near the wall, then a large E×B drift, then a Kelvin-Helmholtz instability, vortices and coalescence. Particle transport to the walls is Bohm-like, for $\omega_{pi} > 2\omega_{ci}$.

B. Plasma transport across B_0 to a wall is studied without and with an active antenna buried in the wall. Particle losses are increased appreciably by the fields of the antenna.

Plasma Wall Physics, Theory and Simulation

A. Small angle Coulomb collisions produce a drag force and diffusion tensor, which are calculated self-consistently. This produces a FP-PIC method. An example is shown for a beam scattering off fixed ions.

B. Implicit particle simulations are discussed for bounded plasma.

C. The effect of source distribution on the sheath potential is discussed.

Code Development

A. Initiation of an effort to simulate a traveling-wave tube is discussed.

B. Substantial progress has been made in making our standard periodic code ES1 work efficiently on a fast PC.

C. Same for our bounded code PDW1.

D. An appreciable factor in B, C, above has been the enhancement of the user interface, especially with access to many windows.

E. A discussion is presented on improving distributing particles across a given distribution in 1d and 2d.

Sheath Seminar Spring 1988

A. The seminar speaker schedule is presented.

B. Some possible characterizations for sheaths are presented.

C. A plasma sheath reference listing is discussed.

FIRST AND SECOND QUARTER PROGRESS REPORT
ON
PLASMA THEORY AND SIMULATION

January 1 to June 30, 1988

Our research group uses both theory and simulation as tools in order to increase the understanding of instabilities, heating, transport, plasma-wall interactions, and large potentials in plasmas. We also work on the improvement of simulation, both theoretically and practically.

Our staff is:

Professor C.K. Birdsall	191M	Cory Hall	643-6631
Principal Investigator			
Dr. William Lawson	199MD	Cory Hall	642-1297
Dr. Ian Morey	187M	Cory Hall	642-3477
Post-doctorates			
Mr. Scott Parker	199MD	Cory Hall	642-1297
Mr. Richard Procassini	199MD	Cory Hall	642-1297
Ms. Julia Little	199MD	Cory Hall	642-1297
Research Assistants(students)			

Our advisers and Associates are:

Dr. Ilan Roth	304	SSL	642-1327
Physicist, Space Science Lab, UCB			
Ms. Lou Ann Schwager		Sandia	
Post doc at Sandia Labs, Livermore			
Dr. Bruce Cohen	L630	LLNL	422-9823
Dr. Alex Friedman	L630	LLNL	422-0827
Dr. A. Bruce Langdon	L472	LLNL	422-5444
Physicists, Lawrence Livermore Natl. Lab			

June 30, 1988

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*** Supported in part by IR&D Grant from LLNL.

**** Supported by ATRTP.

***** Supported by SAIC/DOE

A.

Vortex Formation and Particle Transport in a Cross-Field Plasma Sheath

K. Theilhaber and C.K. Birdsall
Electronics Research Laboratory,
University of California,
Berkeley, CA 94720

Abstract

The time-dependent behavior of a transversely magnetized, two-dimensional plasma-wall sheath has been studied through particle simulations, which have shown that the cross-field sheath develops into a turbulent boundary layer, with strong potential fluctuations and anomalous particle transport. The driving mechanism is the Kelvin-Helmholtz instability, which arises from the sheared particle drifts created near the wall. The sheath acquires an equilibrium thickness $l_x \sim 5\ \rho_i$, and maintains large, long-lived vortices, with amplitudes $\delta\phi \sim -2T_i/e$, which drift parallel to the wall at roughly half the ion thermal velocity. The sheath also maintains a large, spatially-averaged potential drop from the wall to the plasma, in sharp distinction with the unmagnetized sheath, where the plasma potential is *higher* than at the wall. Accompanying the vortices is a spectrum of shorter-wavelength fluctuations which induce an anomalous cross-field transport. A central simulation result is that for $\omega_{pi} \geq 2\omega_{ci}$, the transport scales like Bohm diffusion, a result for which we have a qualitative analytic model.

Submitted to *Physical Review Letters*, June 1988.

B. Theoretical Studies of Edge Physics and Plasma/Wall Interactions*

C.K. Birdsall and W.S. Lawson UCB
D.Harned SAIC San Diego

The proposed work represents a collaboration between SAIC and the University of California, Berkeley to address important issues in the physics of the plasma edge. In particular, state of the art tools will be applied to understanding the coupling between the plasma edge, modeled by a fully kinetic particle code and the plasma center modeled by a semi implicit resistive MHD code. This work is expected to significantly enhance our understanding of divertors and the plasma H mode. In addition, the management plan for this collaboration includes significant interactions with experimental groups involved in these issues. (Excerpt from letter from Dr. D.A. Hitchcock, Fusion Theory Branch, DOE September 14, 1987.)

The original plan was that collaboration between fluid and particle researchers, SAIC and UCB respectively, could lead to substantial advances in understanding magnetized plasmas near walls, such as found in tokamaks, with limiters and diverters. After the original plan, interest at DOE increased in learning more about edge-plasma behavior at and near an ICRF antenna with electrostatic shield, as used for tokamak heating; we added this objective, upgrading the complexity.

The Berkeley effort, with a small doctoral student group, was originally slated to begin May 1, 1987 which became about October 1, 1987. (Support was not received in Berkeley until months later.) We had the good fortune that Dr. William Lawson agreed to stay around a bit after his thesis was completed, in order to start some of the plasma-edge work proposed. The work performed so far, directly on this contract, is mostly by him. Unfortunately, having just begun, we were then told we would only receive six months support, to end April 1, 1988.

ACCOMPLISHMENTS

Our basic machine for doing our computer experiments on a magnetized two-dimensional plasma in contact with a wall is our many-particle code ES2 developed here by Dr. Kim Theilhaber. (We are also developing PDW2, using much of ES2, along the lines of our well known and widely used 1d code, PDW1.) The major recent work with this code, by Dr. Theilhaber, is "Vortex Formation and Particle Transport in a Cross-Field Plasma Sheath" Memorandum No. UCB/ERL M88/21, 20 March 1988, 100 pages, widely circulated. A short version has been submitted to *Physical Review Letters* and a more detailed version is to be submitted to the *Physics of Fluids*.

* This is a progress report only. A final report is being prepared with all of the detailed results.

This report goes into great detail on the sheath formation, edge waves, instability (Kelvin-Helmholtz), vortex formation and coalescence, and anomalous Bohm-like transport of particles to the wall. This effort, which took place over two years, with joint DOE and other support, covers the undriven magnetized plasma sheath very well. It forms one of the starting points for the joint SAIC-UCB Berkeley work, which will now be described.

It is to be noted most firmly that while fluid codes and implicit particle codes may be applied to a magnetized plasma near a wall, such codes are not cabable of resolving all of the detailed physics needed, on the scale length of a Debye length or a gyroradius, in the sheath. The latter physics, which is needed for the transport across the sheath (particles, energy, heat) may only be produced with any amount of trust by explicit particle codes such as we are using. Such physics is needed for these fluxes, as well as the wave and instability descriptions. It is the marriage of particle codes near the wall and fluid codes in the bulk plasma, including the use if implicit coding away from the wall, that is proposed.

UNDRIVEN PLASMA/WALL RESULTS

In the plasma-wall region which we will label the *sheath* , we find an *unusual plasma,* with:

$$\left[\frac{e\phi}{KT_e}\right] \geq 1$$

$n_i \neq n_e$

gradient lengths scaling with λ_D or ρ_i

f(v) non-Maxwellian

momentum reversals (of one species)

surface modes (e.g., Kelvin-Helmholtz waves)

wall charges and currents

wall absorption, reflection, emission (i.e., neutrals)

This *un-plasma* is very different from the usual nearly uniform, nearly quiet, nearly Maxwellian, nearly neutral textbook plasma; the *un-plasma* tends to demand kinetic treatment, using particles.

Four long computer runs were made on the ES2 codes of Theilhaber, starting with an appreciable *vacuum gap* between the metal absorbing wall and the plasma edge. The object was to observe whether there was any appreciable new physics with the plasma initially away from the wall or up against the wall. The latter case was studied in detail by K. Theilhaber, leading to Kelvin-Helmholtz instability, formation of vortices, coalescence, and Bohm-like diffusion to the wall.

The conclusions are that:

(a) the plasma in all runs moves across the vacuum gap of many ρ_e, perhaps $5\rho_i$;

(b) the initial instability is a finite gain ion gyroradius drift wave, developing into what may be a Kelvin-Helmholtz velocity-shear-driven mode.

Hence, as was an objective of these runs, it appears that the plasma tends to fill the space alloted. Therefore, it is quite all right to start studies with the plasma against the wall, to be described now.

THE ANTENNA-DRIVEN PLASMA-WALL RESULTS

In the previous section, we found that the plasma would rather readily close up a gap of a few ion gyroradii, suggesting that it is permissible to start the plasma right up against the wall. Hence, we felt free to start the driven plasma runs without the vacuum gap. Long runs were made, with similar parameters as before.

We found that the electrostatic near field of the ICRF antenna substantially increases the particle flux and energy to the antenna sheild, which, in the laboratory, results in a large increase in sheild sputtering. It is this increase (due to the above and other effects as well) which limits operation in some of the large tokamaks; such effects are well stated in the summary, by R.E. Aamodt, of the ICRF/Edge Physics Workshop, Boulder CO, March 30 — April 1, 1988.

Obviously, many more runs and some more diagnostics are called for, as well as invention in the electrostatic shield and antenna configuration, so as to alter (reduce) the wall sputtering.

A. A Proposed Particle-In-Cell Method for Modeling Small Angle Coulomb Collisions in Plasmas

S. E. Parker
Electronics Research Laboratory
Univ. of California, Berkeley

December 6, 1988

1 Introduction

We are proposing a computational method to self-consistently model small angle collisional effects. This method may be used in standard Particle-In-Cell (PIC) plasma simulations to include collisions, or as an alternative to solving the Fokker-Planck (FP) equation using finite difference methods. The distribution function is represented by a large number of particles. The particle's velocities change as a function of the drag force, and the diffusion in velocity is represented by a Wiener process. This is similar to previous Monte-Carlo methods [1,2], except we calculate the drag force and diffusion tensor self-consistently. The particles are weighted to a grid in velocity space and the associated "Poisson's equations" are solved for the Rosenbluth potentials. First the approximation for small angle Coulomb collisions is discussed. Next, the FP-PIC collision method is outlined. Then we show a test of the particle advance modeling an electron beam scattering off a fixed ion background.

2 Small angle Coulomb collisions

The FP equation for describing small angle Coulomb collisions can be solved numerically using finite difference techniques. An alternate method is to follow the evolution of a large number of particles. Using the notation of Trubnikov [3], an infinitestimal "cloud" of test particles can be represented by the quantities $< \Delta v_i >$ and $< \Delta v_i \Delta v_j >$ to the same order of accuracy as the FP equation[3]. These are defined as:

$$< \Delta v_i > \equiv \frac{d\bar{v}_i(t)}{dt} \tag{1}$$

$$< \Delta v_i \Delta v_j > \equiv \frac{d}{dt}\left\{\overline{(v_i - \bar{v}_i)(v_j - \bar{v}_j)}\right\} \tag{2}$$

The bars represent averages. These may be interpreted as ensemble averages over many initial states, or as a straight average by letting the number of particles in a cloud, N, get large, $N \to \infty$. We follow the second interpretation allowing a simpler simulation method. Then $\bar{v}_i$ is the average velocity of the local cloud. The i and j subscripts are the 3 velocity components in cartesian coordinates, $(v_1 = v_x, v_2 = v_y, v_3 = v_z)$. $< \Delta v_i >$ is the drag force felt on an average particle in the cloud. $< \Delta v_i \Delta v_j >$ is the spreading or diffusion in velocity space of the cloud.

Numerically we change the velocity of the particles by using the following equation [4]:

$$\Delta v_i(t^n) = < \Delta v_i > \Delta t + B_{ij} \Delta W_i(t^n) \tag{3}$$

where B_{ij} satisfies:

$$B_{ik} B_{jk} = < \Delta v_i \Delta v_j > \tag{4}$$

$\Delta W_i(t^n) = W_i(t^n) - W_i(t^{n-1})$ and $W_i(t)$ is a vector function that represents a Wiener process (or Brownian motion) having the following properties $< W_i(t) >= 0$ and $< W_i(t) W_j(t) >= t\delta_{ij}$. B and W are not unique and only need to satisfy the above criteria. In our model we choose:

$$\Delta W_i(t^n) = \sqrt{\Delta t} R_i \tag{5}$$

where R_i are independent random numbers satisfying $< R_i >= 0$, and $< R_i R_j >= \delta_{ij}$. We

have found a a matrix B such that $BB^\tau = A$, given by:

$$B_{ij} = \begin{pmatrix} \sqrt{A_{11}} & 0 & 0 \\ \frac{A_{12}}{\sqrt{A_{11}}} & \sqrt{A_{22} - \frac{A_{12}^2}{A_{11}}} & 0 \\ \frac{A_{13}}{\sqrt{A_{11}}} & \frac{A_{23}}{B_{22}} - \frac{A_{12}A_{13}}{A_{11}B_{22}} & \sqrt{A_{33} - \frac{A_{13}^2}{A_{11}} - B_{32}^2} \end{pmatrix} \tag{6}$$

By letting $A_{ij} =< \Delta v_i \Delta v_j >$ we note that equation (6) satisfies eq. (4) and can therefore use this B in equation (3).

The "field" quantities are obtained from two functions ϕ and ψ, the Rosenbluth potentials, which in turn are solved by the two Poisson's Equations [3]:

$$\nabla^2 \phi^\beta = f^\beta(v_i) \tag{7}$$

$$\nabla^2 \psi^\beta = \phi^\beta(v_i) \tag{8}$$

where β is the superscript representing the field species, $\beta = (i, e)$. Then $< \Delta v_i >$ and $< \Delta v_i \Delta v_j >$ are obtained in terms of the Rosenbluth potentials using the following equation (see reference [1] for a derivation):

$$< \Delta v_i >^\alpha = -\sum_\beta \left(1 + \frac{m_\alpha}{m_\beta}\right) L^{\alpha/\beta} \frac{\partial \phi_\beta}{\partial v_i} \tag{9}$$

$$< \Delta v_i \Delta v_j >^\alpha = -2 \sum_\beta L^{\alpha/\beta} \frac{\partial^2 \psi_\beta}{\partial v_i \partial v_j} \tag{10}$$

$L^{\alpha/\beta} = \lambda \left(\frac{4\pi e_\alpha e_\beta}{m_\alpha}\right)^2$ is a constant (given here in cgs units), and λ is the Coulomb logarithm. α and β represent the test and field particles, respectively (e.g. $\alpha = (i, e)$ and $\beta = (i, e)$).

3 An Outline of the Method

For the particle advance we start with the simplest scheme [1,2] similar to Euler's method except there is an added diffusion term (the last term on the right):

$$v_i^{n+1} = v_i^n + < \Delta v_i >^n \Delta t + B_{ij}^n \sqrt{\Delta t} R_j \tag{11}$$

Where R_j are independent random numbers having the two properties given above. Following the methodology of PIC simulation, we weight the particles to a grid, calculate the field quantities, advance the particles and repeat the process. The basic algorithm is as follows:

Step 1. Weight the particles to a grid in velocity space using linear interpolation.

Step 2. Solve equations (6) and (7) on the grid for ϕ and ψ.

Step 3. Obtain $< \Delta v_i >^n$ and B_{ij}^n on the grid using equations (6), (9) and (10).

Step 4. For each particle, obtain $< \Delta v_i >^n$ and B_{ij}^n by interpolating from the grid to the particle location v_i. Then update the velocity using equation (11).

4 Solution to the "Field" Equations

In step 2. above, we need to solve Poisson's equation twice in 3 dimensions. That is, $\nabla^2 u = s$, where $u = (\phi, \psi)$ and $s = (f, \phi)$. To simplify the problem, we assume azimuthal symmetry in v_x-v_y, and work in a 2 dimensional cylindrical geometry (r,z) where $r^2 = v_x^2 + v_y^2$ and $z = v_z$. Since the system is independent of the spatial coordinates we define: $x = v_x$, $y = v_y$ and $z = v_z$ *without* confusion. A finite Fourrier transform (FT) is made in z, then we can solve for each k using the following finite difference scheme [5]:

$$\frac{2}{\Delta r \delta r_j^2}\left\{r_{j+1/2}(\hat{u}_{j+1} - \hat{u}_j) + r_{j-1/2}(\hat{u}_j - \hat{u}_{j-1})\right\} - \kappa^2 \hat{u}_j = \hat{s}(r,k) \tag{12}$$

where:

$$r_{j+1/2} \equiv r_j + \frac{1}{2}\Delta r \tag{13}$$

$$\delta r_j^2 \equiv r_{j+1/2}^2 - r_{j-1/2}^2 \tag{14}$$

κ is the finite difference analog to k[5]:

$$\kappa \equiv \left(\frac{2}{\Delta z}\right) sin\left(\frac{k\Delta z}{2}\right) \tag{15}$$

This tridiagonal system is solved for each k, and an inverse FT is made to get u, $u(r,z) = FT^{-1}(\hat{u}(r,k))$.

The next step is to calculate $< \Delta v_i >$ and $< \Delta v_i \Delta v_j >$ from ϕ and ψ. Using the change of coordinates and the assumption of azimuthal symmetry we obtain the following:

$$\frac{\partial \phi}{\partial v_i} = (\frac{x}{r}\phi_r, \frac{y}{r}\phi_r, \phi_z) \tag{16}$$

$$\frac{\partial^2 \psi}{\partial v_i \partial v_j} = \begin{pmatrix} \frac{x^2}{r^2}\psi_{rr} + (\frac{1}{r} - \frac{x^2}{r^3})\psi_r & \frac{xy}{r^2}\psi_{rr} - \frac{xy}{r^3}\psi_r & \frac{x}{r}\psi_{rz} \\ \frac{xy}{r^2}\psi_{rr} - \frac{xy}{r^3}\psi_r & \frac{y^2}{r^2}\psi_{rr} + (\frac{1}{r} - \frac{y^2}{r^3})\psi_r & \frac{y}{r}\psi_{rz} \\ \frac{x}{r}\psi_{rz} & \frac{y}{r}\psi_{rz} & \phi_{zz} \end{pmatrix} \tag{17}$$

We approximate $\phi_r, \phi_z, \psi_r, \psi_{rr}$, and ψ_{zz}, by central differencing ϕ and ψ. Then use equations (8) and (9) to get $< \Delta v_i >$ and $< \Delta v_i \Delta v_j >$ on the grid.

5 Test of the particle advance

As an initial test of the particle advance, eq. (11), we model an electron beam scattering off infinitely massive cold ions. For this test case $< \Delta v_i >$ and $< \Delta v_i \Delta v_j >$ are given by the two simple analytic expressions:

$$< \Delta v_i > = -C\frac{v_i}{v^3} \tag{18}$$

$$< \Delta v_i \Delta v_j > = C\left(\frac{\delta_{ij}}{v} - \frac{v_i v_j}{v^3}\right) \tag{19}$$

where $C = \dfrac{n_\beta L^{\alpha/\beta}}{4\pi}$. We do not calculate the field quantities on the grid, but rather, use equations (18) and (19) explicitly. This gives a good test of the particle advance, equation (11). We expect the total x-momentum to be given by:

$$p_{x,total}(t) = p_{x,total}(t=0)exp\left(\frac{-C}{v_0^3}t\right) \tag{20}$$

where v_0 is the initial beam velocity. For the run shown: $C = 1$, $v_0 = 1$, $v_{Te} = 0.01$ and $\Delta t = 0.001$. Figure 1 shows the initial beam. Figure 2. is the total x-momentum vs. time. Figures 3 and 5 show snapshots of (v_x, v_y). Figure 4 and 6 also are (v_x, v_y) scatter plots but, a small slice in v_z, $|v_z| < 0.1$. The energy error is less than 0.2 percent and the momentum error is less then 1 percent.

6 Future Work

We are developing a code to test the FP-PIC method outlined above. In the future we plan to implement the algorithm in a 1d 3v electrostatic code to study combined collective electrostatic and collisional effects. In this case we would assume spatial homogeneity for

calculating the collisional terms. If this assumption is not valid one would have to partition $f(v)$ into spatial regions j, and calculate $f_j(v)$ for each spatial region. One would have to ensure that enough particles in each spatial zone to adequately fill out the distribution function.

In addition, we need to address the accuracy and stability issues associated with this method, and study the feasibility of using a more accurate particle advancing scheme than that given by equation (10).

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[3] "Particle Interactions in a Fully Ionized Plasma", B. A. Trubnikov, p.105-204, *Reviews of Plasma Physics*, M. A. Leontovich ed., Consultants Bureau, New York, 1965.

[4] *Handbook of Stochastic Methods*, C. W. Gardiner, Springer-Verlag, New York 1985.

[5] *Plasma physics via computer simulation*, C. K. Birdsall and A. B. Langdon, McGraw-Hill, New York, 1985.

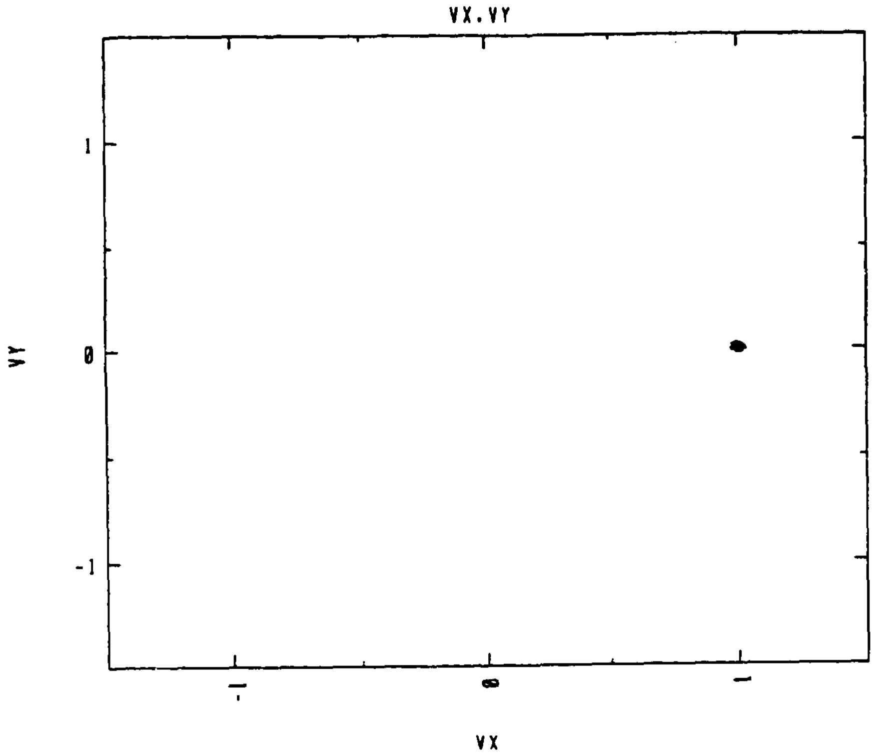


Figure 1: (v_x, v_y) for the beam at t=0.

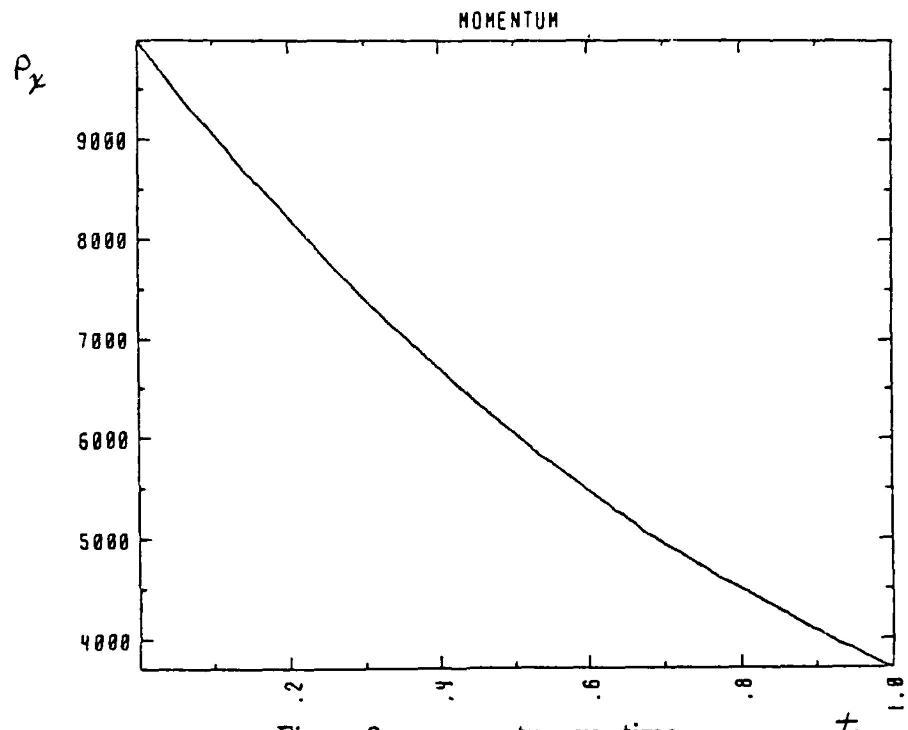


Figure 2: x-momentum vs. time.

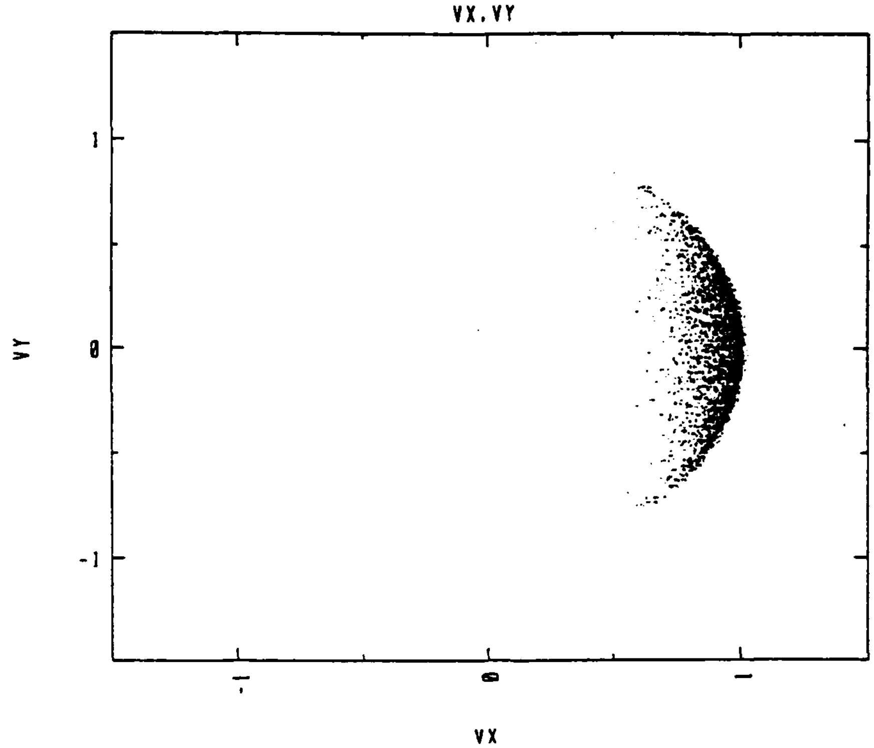


Figure 3: (v_x, v_y) for the beam at t=0.1.

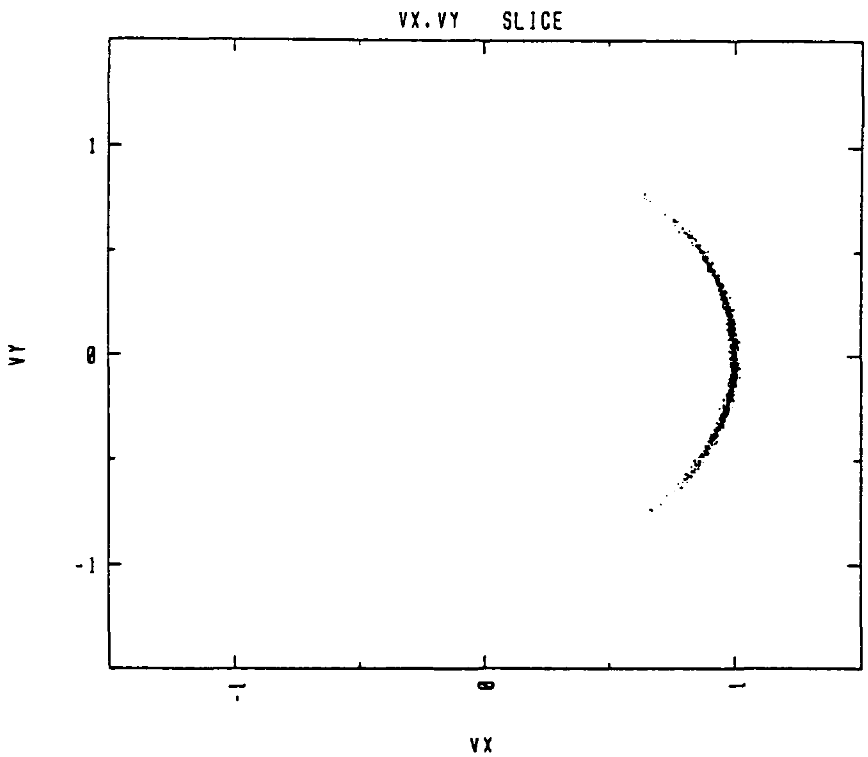


Figure 4: Slice in v_z of beam at t=0.1, $|v_z| < 0.1$.

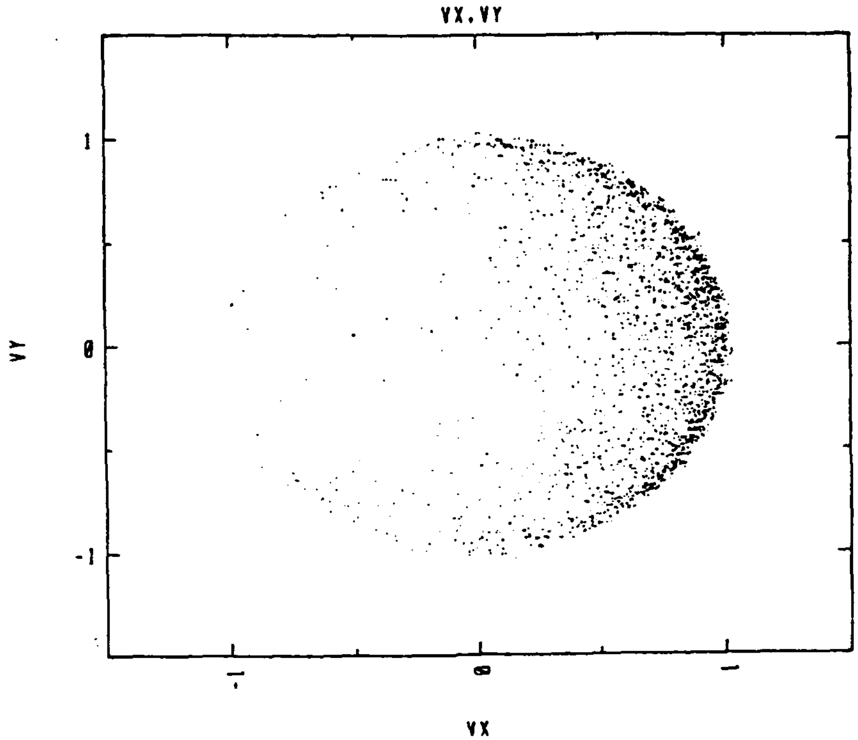


Figure 5: (v_x, v_y) for the beam at t=1.

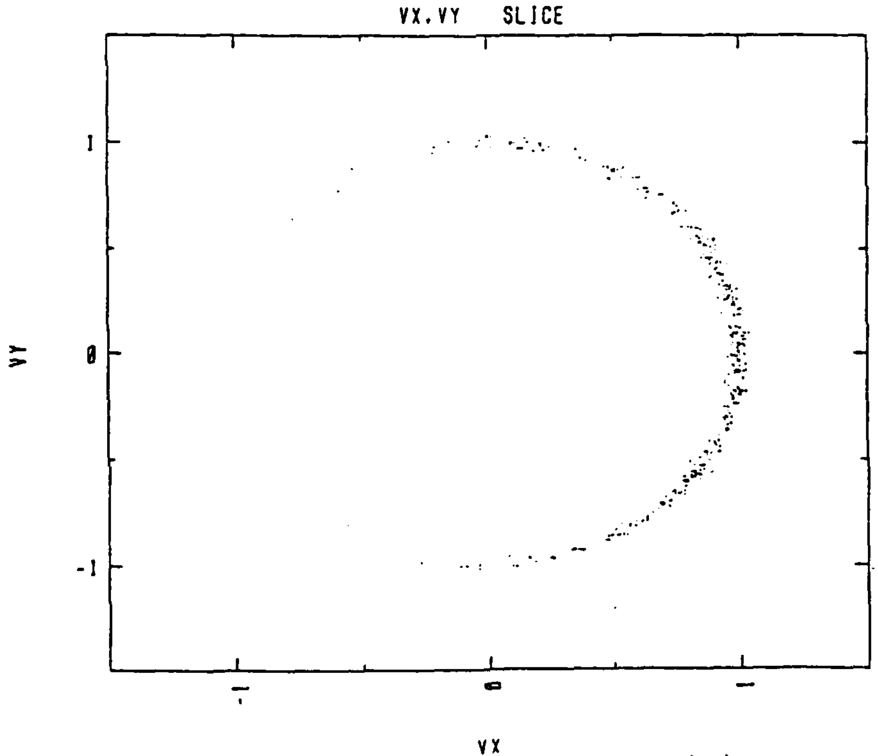


Figure 6: Slice in v_z of beam at t=1, $|v_z| < 0.1$.

B. # Direct-Implicit Particle Simulations of Collisionless and Collisional Bounded Plasmas.

R. J. Procassini and C. K. Birdsall

and

B. I. Cohen (Lawrence Livermore National Laboratory)

Direct-implicit particle-in-cell simulation methods are applied to the study of bounded plasmas systems, with the goal of obtaining a self-consistent description of plasma-surface interactions in the region near the divertor plate of a tokamak. Several important phenomena in this region of the scrape-off-layer (ionization of recycled neutral gas, charged/neutral collisions, etc.) have characteristic time and space scales that are long compared to the electron plasma frequency and Debye length. Direct-implicit methods provide an efficient means for accurately reproducing these effects, without having to resolve electron relevant time and space scales. We will present the results of large $\omega_{pe}\Delta t$, $\Delta x/\lambda_{De}$ implicit particle simulations of a bounded plasma system in both the collisionless and collisional limits. These simulations are not capable of resolving λ_{De}, hence the sheath width is based upon the grid spacing. The total potential drop through the presheath and sheath in this implicit limit will be investigated, and compared to theoretical predictions.

C. # Self-Consistent Calculations of the Sheath and Presheath in a Bounded, Collisionless Plasma with Different Source Distributions.

R. J. Procassini and C. K. Birdsall

and

B. I. Cohen (Lawrence Livermore National Laboratory)

Two different models for the calculation of the sheath and presheath in a bounded, collisionless plasma have recently been proposed by Emmert, et. al. [1] and Bissell and Johnson[2]. These models differ in the choice of the source distribution function for the ions. Emmert uses $f_s(v) = |v| f_M(v)$, while Bissell and Johnson use $f_s(v) = f_M(v)$, where $f_M(v)$ is a Maxwellian. Both models assume Boltzmann electrons. We employ particle-in-cell techniques to study the effect of these different source distribution functions on the sheath and presheath, without making the assumption of Boltzmann electrons. Our electron distribution function is therefore cut-off by the collector sheath potential. Self-consistent potential profiles for each type of source distributions will be presented.

[1] G. A. Emmert, et. al. , *Phys. Fluids*, **23**, 803 (1980).

[2] R. C. Bissell and P. C. Johnson, *Phys. Fluids*, **30**, 779 (1987).

SECTION III: CODE DEVELOPMENT

A.

Particle-Circuit Simulation of Traveling-Wave-Tubes

Ian J. Morey

A code is being developed to simulate a traveling-wave-tube. It uses the particle-in-cell technique to model the electron beam, combined with a leapfrog scheme for modeling the coupled transmission line. An advantage of the particle-in-cell technique is the ease with which space-charge effects can be modelled. Since the code was initially intended to be used as a teaching aid, it is being developed to run on a PC with many diagnostics of the interaction displayed graphically. However, it is believed that the code could become a useful research tool if run on a faster computer. At present, many parameters can be changed while the code is running, such as: source, line and load impedances, coupling between the beam and the transmission line, space-charge effects and the input signal amplitude. Other parameters can only be changes at the start of each simulation, such as: beam energy, beam current, tube length and wave velocity.

B.

ES1 on IBM-Compatible Personal Computers

John P. Verboncoeur

The Electrostatic 1 Dimensional Periodic Plasma Simulation (ES1) code originated by A. Bruce Langdon[1] in FORTRAN on the MFE Cray computers and subsequently translated into C by T. Lasinski has been ported to the MS-DOS environment. The PC version of ES1 adds several features to the Cray version which are described below.

The MS-DOS version of ES1 can simulate periodic plasmas comprised of up to 8 different species. The total number of particles for unmagnetized simulation is 16000, and up to 8000 for magnetized species. PC ES1 has the capability to weight charge to up to 8192 grid points using zero order (nearest grid point) or 1st order (particle-in-cell) weighting schemes. The code can run up to 1024 timesteps, storing electrostatic, kinetic, total, and Fourier-decomposed mode energies. All the standard input parameters of ES1 are supported (except plot intervals; PC ES1 plots diagnostics interactively with a keypress). The input parameters are contained in an input deck similar to the namelist input deck of the Cray version. PC ES1 is capable of moving 1000 particles per second on an IBM AT, 2000 on an IBM PS/2 Model 60, and 4000 on a 20 MHz 80386 PC.

Accessibility. The MS-DOS version of ES1 runs on any IBM-compatible PC, AT, or PS/2. The proliferation of these machines enables almost any student or researcher to run and modify ES1. We have already distributed over 50 copies of the code, and it has been used in mini-courses and offsite demonstrations. The complete source code for ES1 is distributed with the program to facilitate additional diagnostics, new particle pushers and field solvers, etc.

Phase-Space Animation. PC ES1 displays a real-time phase space animation with each species in a different color. Using the animation, one can observe one dimensional collisions, accelerations, and particle trapping. PC ES1 can also leave traces along the trajectories of the particles in phase space.

1 C. K. Birdsall and A. B. Langdon, *Plasma Physics Via Computer Simulation* (1985)

C.

PDW1 on IBM Compatible Personal Computers

John P. Verboncoeur

The Plasma Device Workshop One Dimensional Bounded Plasma Simulation (PDW1) is being ported to the MS-DOS environment. The original code, due to William S. Lawson[2], was written in FORTRAN for the MFE Cray computers. In 1986, Tom L. Crystal translated the FORTRAN source code into C. We now intend to create a version of PDW1 on the PC analogous to PC ES1. The initial version will be completed in the 4th quarter of calendar 1988.

2 William S. Lawson, *PDW1 User's Manual*, Memorandum UCB/ERL M84/37 (1984).

D.

ES1 for Windows

John P. Verboncoeur and Vahid Vahedi

Similar to PC ES1, Windows ES1 adds a number of features. Using the multiple display windows capabilities of Windows, Windows ES1 can display any or all of the diagnostics concurrently with the phase space animation. The diagnostics are updated with each step of the animation, resulting in real-time display of space-charge density, fields, etc. The windows can be sized and moved to suit the user's needs. The mouse-driven graphical interface is well suited to the beginner or casual user.

In addition, Windows ES1 adds pop up menus for diagnostics as well as a menu-driven help system. We expect to submit Windows ES1 in the 4th quarter of calendar 1988, satisfying the requirements of the IBM Windows Software Development Contract.

E. The Construction of Distribution Functions

Communication from Dr. Andrew Zachary* to B.I. Cohen LLNL

Your notes on the construction of the "true" distribution function are much appreciated. As we have already discussed, one possible method of construction f requires uniform binning in anonuniform variable. For example, to determine $f(v_\perp)\,v_\perp dv_\perp$, the bins are distributed uniformly in $v_\perp^2$. Similarly, for $f(v)\,v^2 dv$, the bins are uniformly distributed in v^3. Your notes and the different figures in TESS show that you have adopted this approach. However, nonuniform variables require an unacceptable number of bins whenever the particles have a large range in velocity. In my cosmic ray simulations, the particles have momenta which lie in the interval $(0, 200)$, so I cannot use your technique.

There is an alternative – bin the particles uniformly in a uniform variable, and then try to "undo" the binning. For convenience, let me use v to denote the velocity variable, and suppose I have a series of uniformly spaced velocity bins v_i, $i = 1, \ldots, N$. For each point v_i, I have an integral equation for f of the form

$$F(v_i) = \int_{v_{i-1}}^{v_{i+1}} S_{1D}(v - v_i) f(v) v^\alpha dv. \tag{1}$$

Here, $F(v_i)$ denotes the value returned by the usual distribution function routine as the estimate of the distribution function at v_i. Also, $S_{1D}(v - v_i)$ is the usual one-dimensional linear weight function

$$S_{1D}(v - v_i) = \begin{cases} \dfrac{1}{\Delta v}(v - v_{i-1}) & v_{i-1} \le v \le v_i \\ \dfrac{1}{\Delta v}(v_{i+1} - v) & v_i \le v \le v_{i+1} \end{cases},$$

which represents the shape of an individual particle. Furthermore, v^α is the velocity space volume factor, where

$$\alpha = \begin{cases} 0 & \text{if } v = v_x, v_y, v_z, v_\parallel \\ 1 & \text{if } v = v_\perp \\ 2 & \text{if } v = |\mathbf{v}| \end{cases}.$$

* The University of Chicago
Department of Astronomy and Astrophysics
5640 South Ellis Avenue
Chicago, Illinois, 60637

As it stands, Eq. 1 is intractable without some method of approximating $f(v)$. After some experimentation, I believe that the "best" solution involves approximating f on the interval (v_{i-1}, v_{i+1}) by a piecewise linear function

$$f(v) = \begin{cases} f(v_{i-1}) + \frac{1}{\Delta v}(f(v_i) - f(v_{i-1})) & v_{i-1} \le v \le v_i \\ f(v_{i+1}) - \frac{1}{\Delta v}(f(v_i) - f(v_{i+1})) & v_i \le v \le v_{i+1} \end{cases}.$$

This approximation places no constraints on the function form of f, although it does require that $df/dv \gg d^2f/dv^2$ in each subinterval (v_i, v_{i+1}). Except for possibly pathological distribution functions, the piecewise linear approximation seems a reasonable choice.

Now the integral in Eq. 1 can be done in closed form. After some tedious algebra, the result is a tridiagonal set of coupled equations

$$F(v_i) = C_{i,i-1}f(v_{i-1}) + C_{i,i}f(v_i) + C_{i,i+1}f(v_{i+1})$$

where

$$\begin{aligned} C_{i,i-1} &= \frac{\Delta v}{6}\left(\frac{v_i + v_{i-1}}{2}\right)^{\alpha} + \frac{(\Delta v)^3}{120}\delta_{\alpha,2} \\ C_{i,i} &= \frac{2\Delta v}{3}v_i{}^{\alpha} + \frac{(\Delta v)^3}{15}\delta_{\alpha,2} \\ C_{i,i+1} &= \frac{\Delta v}{6}\left(\frac{v_i + v_{i+1}}{2}\right)^{\alpha} + \frac{(\Delta v)^3}{120}\delta_{\alpha,2} \end{aligned}.$$

The symbol $\delta_{\alpha,2}$ is the usual Kronecker delta.

At the upper and lower boundaries, the coefficients $C_{i,i}$ are modified as

$$C_{1,1} = \begin{cases} \frac{\Delta v}{6} & \alpha = 0 \\ \frac{1}{3}v_1\Delta v + \frac{(\Delta v)^2}{12} & \alpha = 1 \\ \frac{\Delta v}{3}\left(v_1^2 + \frac{1}{2}v_1\Delta v + \frac{(\Delta v)^2}{10}\right) & \alpha = 2 \end{cases}$$

$$C_{N,N} = \begin{cases} \frac{\Delta v}{6} & \alpha = 0 \\ \frac{1}{3}v_N\Delta v + \frac{(\Delta v)^2}{12} & \alpha = 1 \\ \frac{\Delta v}{3}\left(v_N^2 - \frac{1}{2}v_N\Delta v + \frac{(\Delta v)^2}{10}\right) & \alpha = 2 \end{cases}$$

My test calculations show that the resulting set of tridiagonal equations are stable and robust over a wide range of velocities. Furthermore, the reconstructed distribution function seems very close to the expected "true" distribution.

I have sufficient confidence in this approach that I have extended it to handle the reconstruction of a 2D distribution function. Suppose I need to reconstruct $f(v_\parallel, v_\perp)$ or $f(\mu, v)$ from the moments $F(v_\parallel, v_\perp)$ or $F(\mu, v)$. Again, assume that F is known on a set of uniformly spaced mesh points (x_i, y_j), with $i = 1, \ldots, N$ and $j = 1, \ldots, M$. The 2D equivalent of Eq. 1 is

$$F(x_i, y_j) = \int_{x_{i-1}}^{x_{i+1}} \int_{y_{j-1}}^{y_{i+1}} S_{2D}(x - x_i, y - y_j) f(x, y) x^\beta y^\alpha dx dy. \qquad (2)$$

I am going to simplify the problem by assuming that x is "unweighted", *i.e.* that $\beta = 0$. Fortunately, $S_{2D}(x - x_i, y - y_j)$ factors into two parts

$$S_{2D}(x - x_i, y - y_j) = S_{1D}(x - x_i) S_{1D}(y - y_j).$$

Then the integral in Eq. 2 couples the value of f at (x_i, y_j) to the values of f at the 8 nearest neighboring points.

$i-1, j+1$		$i, j+1$		$i+1, j+1$
	②		①	
$i-1, j$		i, j		$i+1, j$
	③		④	
$i-1, j-1$		$i, j-1$		$i+1, j-1$

In the spirit of the one-dimensional technique, I approximate $f(x, y)$ by a piecewise bilinear function. Explicitly, in each region shown above, I write

$$\begin{aligned}
W f_1(x, y) &= (x_{i+1} - x)(y_{j+1} - y) f(x_i, y_j) + (x_{i+1} - x)(y - y_j) f(x_i, y_{j+1}) \\
&\quad + (x - x_i)(y_{j+1} - y) f(x_{i+1}, y_j) + (x - x_i)(y - y_j) f(x_{i+1}, y_{j+1}) \\
W f_2(x, y) &= (x - x_{i-1})(y_{j+1} - y) f(x_i, y_j) + (x - x_{i-1})(y - y_j) f(x_i, y_{j+1}) \\
&\quad + (x_i - x)(y_{j+1} - y) f(x_{i-1}, y_j) + (x_i - x)(y - y_j) f(x_{i-1}, y_{j+1}) \\
W f_3(x, y) &= (x - x_{i-1})(y - y_{j-1}) f(x_i, y_j) + (x - x_{i-1})(y_j - y) f(x_i, y_{j-1}) \\
&\quad + (x_i - x)(y - y_{j-1}) f(x_{i-1}, y_j) + (x_i - x)(y_j - y) f(x_{i-1}, y_{j-1}) \\
W f_4(x, y) &= (x_{i+1} - x)(y - y_{j-1}) f(x_i, y_j) + (x_{i+1} - x)(y_j - y) f(x_i, y_{j-1}) \\
&\quad + (x - x_i)(y - y_{j-1}) f(x_{i+1}, y_j) + (x - x_i)(y_j - y) f(x_{i+1}, y_{j-1})
\end{aligned}$$

where $W = \Delta x \Delta y$. After some more tedious algebra, the integrals in each region can again be done in closed form. The results are

$$\alpha = 1$$

$$F_1(x_i,y_j) = \frac{W}{9}\left[(y_j + \frac{\Delta y}{4})(f_{i,j} + \frac{1}{2}f_{i+1,j}) + \frac{1}{2}(y_j + \frac{\Delta y}{2})(f_{i,j+1} + \frac{1}{2}f_{i+1,j+1})\right]$$

$$F_2(x_i,y_j) = \frac{W}{9}\left[(y_j + \frac{\Delta y}{4})(f_{i,j} + \frac{1}{2}f_{i-1,j}) + \frac{1}{2}(y_j + \frac{\Delta y}{2})(f_{i,j+1} + \frac{1}{2}f_{i-1,j+1})\right]$$

$$F_3(x_i,y_j) = \frac{W}{9}\left[(y_j - \frac{\Delta y}{4})(f_{i,j} + \frac{1}{2}f_{i-1,j}) + \frac{1}{2}(y_j - \frac{\Delta y}{2})(f_{i,j-1} + \frac{1}{2}f_{i-1,j-1})\right]$$

$$F_4(x_i,y_j) = \frac{W}{9}\left[(y_j - \frac{\Delta y}{4})(f_{i,j} + \frac{1}{2}f_{i+1,j}) + \frac{1}{2}(y_j - \frac{\Delta y}{2})(f_{i,j-1} + \frac{1}{2}f_{i-1,j-1})\right]$$

and

$$\alpha = 2$$

$$\begin{aligned} F_1(x_i,y_j) = \frac{W}{9}\Bigg[&\left(y_j^2 + \frac{y_j\Delta y}{2} + \frac{(\Delta y)^2}{10}\right)(f_{i,j} + \frac{1}{2}f_{i+1,j}) \\ &+ \frac{1}{2}\left(y_j^2 + y_j\Delta y + \frac{3(\Delta y)^2}{10}\right)(f_{i,j+1} + \frac{1}{2}f_{i+1,j+1})\Bigg] \end{aligned}$$

$$\begin{aligned} F_2(x_i,y_j) = \frac{W}{9}\Bigg[&\left(y_j^2 + \frac{y_j\Delta y}{2} + \frac{(\Delta y)^2}{10}\right)(f_{i,j} + \frac{1}{2}f_{i-1,j}) \\ &+ \frac{1}{2}\left(y_j^2 + y_j\Delta y + \frac{3(\Delta y)^2}{10}\right)(f_{i,j+1} + \frac{1}{2}f_{i-1,j+1})\Bigg] \end{aligned}$$

$$\begin{aligned} F_3(x_i,y_j) = \frac{W}{9}\Bigg[&\left(y_j^2 - \frac{y_j\Delta y}{2} + \frac{(\Delta y)^2}{10}\right)(f_{i,j} + \frac{1}{2}f_{i-1,j}) \\ &+ \frac{1}{2}\left(y_j^2 - y_j\Delta y + \frac{3(\Delta y)^2}{10}\right)(f_{i,j-1} + \frac{1}{2}f_{i-1,j-1})\Bigg] \end{aligned}$$

$$\begin{aligned} F_4(x_i,y_j) = \frac{W}{9}\Bigg[&\left(y_j^2 - \frac{y_j\Delta y}{2} + \frac{(\Delta y)^2}{10}\right)(f_{i,j} + \frac{1}{2}f_{i+1,j}) \\ &+ \frac{1}{2}\left(y_j^2 - y_j\Delta y + \frac{3(\Delta y)^2}{10}\right)(f_{i,j-1} + \frac{1}{2}f_{i+1,j-1})\Bigg] \end{aligned}$$

When the four terms are added together, the result is

$$\begin{aligned} F(x_i,y_j) = {} & \mathcal{A}_{i,j}f_{i-1,j+1} + \mathcal{B}_{i,j}f_{i,j+1} + \mathcal{C}_{i,j}f_{i+1,j+1} + \mathcal{D}_{i,j}f_{i-1,j} + \mathcal{E}_{i,j}f_{i,j} \\ & + \mathcal{F}_{i,j}f_{i+1,j} + \mathcal{G}_{i,j}f_{i-1,j-1} + \mathcal{H}_{i,j}f_{i,j-1} + \mathcal{I}_{i,j}f_{i+1,j-1} \end{aligned}$$

Now, if I let $\ell = (j-1)N + i$, then I can transform these equations a single, giant matrix equation. This matrix has 9 non-zero diagonals, and each row has the pattern: 3 non-zero elements, followed by $N-1$ zeros, followed by 3 non-zero elements, then another $N-1$ zeros, 3 more non-zero elements, and then the rest all zeros. I have not yet written the routines to solve the resulting sparse set of linear equations. I am talking with some of the math people at MFE, and hope to have the routines ready by the end of the month.

A. University of California, Berkeley CA 94720 5 May 1988

FINAL SCHEDULE for seminar on

PLASMA SHEATHS AND OTHER BOUNDARY LAYERS

This is an interdisciplinary plasma seminar, sponsored by faculty from the departments of Chemical Engineering, Electrical Engineering and Computer Science, Mechanical Engineering, Nuclear Engineering, and Physics. The organizer is Prof. Charles K. Birdsall, EECS, 191M Cory Hall, (415)643-6631. See the initial flyer for more description. Partial support for this seminar comes from a MICRO project with Varian gift, gratefully acknowledged.

Meetings are Mondays, 2-4pm, January 25 through May 9, in 240 Bechtel Engr (No meetings on February 15 or March 28 due to University holidays.)

The order being followed is roughly 1/3 DC sheaths, 1/3 RF driven plasmas, and 1/3 magnetized plasmas and 2d effects. A major objective is to make ties among theories, experiments, and simulations for various models; the references will have labels T,E,S.

January 25 Prof. C.K. Birdsall, EECS, UCB, review of the classicl theory and experiments on the plasma potential of L. Tonks, I. Langmuir,
T,E "A General Theory of the Plasma of an Arc", Physical Review 34, September 15, 1929, pp. 876-922.

February 1 Dr. Lou Ann Schwager, Sandia (Livermore) and UCB, a review of the works of the 1960's and 1970's leading up to her own theory and simulations (dissertation, 1987),
T,E "Kinetic Model of the Collisionless Plasma Region, Applied to Thermionic Convertors, Q-Machines, and the Boundary Layer of a Maxwellian Plasma (covering both source and collector sheaths)"

February 8 Julia Little, Math UCB and Scott Parker, NE UCB will review further theory and computation, building on the two previous talks, where more complete solutions have been computed, from

the following references:

T S. A. Self, "Exact Solution of the Collisionless Plasma Sheath Equation", Physics of Fluids **6**, December 1963, pp. 1762-1768.

T S. A. Self, "Asymptotic Plasma and Sheath Representations for Low-Pressure Discharges" Jour. Appl. Phys. **36**, Feb.1965,456-459

T,E D. Dunn and S. A. Self, Static Theory of Density and Potential Distribution in a Beam-Generated Plasma" Jour. Appl. Phys. **35**, January 1964, pp. 113-122.

T J. Parker, "Collisionless Plasma Sheath in Cylindrical Geometry" Phys. Fluids. **6**, 1963, pp 1657-1658.

T G. Emmert, R. Wieland, A. Mense, and J. Davidson, "Electric Sheath and Presheath in a Collisionless, Finite Ion Temperature Plasma", Phys. Fluids **23**, April 1980, pp. 803-812.

T,E S. Meassick, M.H. Cho, and N. Hershkowitz, "Measurement of Plasma Presheath" IEEE Trans. on Plasma Science, **PS-13**, April 1985, pp.115-119.

February 22 Leo Eskin, Stanford University (with Prof. S. A. Self) "Electrode
T Plasma Boundary Layer and Current Transfer for a Weakly-Ionized, Collision Dominated Thermal Plasma, Including Generation"

February 29 Blake Wood, EECS Dept., UCB. Review of boundary conditions on
T ion energy and velocity entering collector sheath, starting with Bohm (1949), continuing through Harrison and Thompson (1959), Hall, Hall and Bernstein, Chodura, Franklin, Lamb (1987).

March 7 (a) Prof. M. A. Lieberman, EECS Dept., UCB. "Analytic Solution for
T RF Driven Sheaths"
T (b) Chris Goedde, Physics, UCB. " Calculations of RF Sheath Electron Heating"

March 14 Dr. Ian Brown, LBL. Plasma Immersion IOn Implantation Sheaths

March 21 Ajit Paranjpe, Stanford University (with Prof. S. A. Self), "RF
T Plane Parallel Low-Pressure Discharge in SF_6, Solving the Boltzmann Equation for $f_e(t)$ and Calculating the Charged-Particle Balance Equations for Electrons, + and - Ions".

April 4 (a) Prof. David Graves, Chem. Engr. UCB, "Review of Modeling for Short
T, S Mean Free Path RF Discharges", including works of Graves, Godyak.
(b) Prof. Sidney Self, ME Dept., Stanford, "Boundary Conditions for
T Continuum Equations of the Diffusion Type, with and without Migration due to Body Forces" with Applications to Diffusion in Neutral Gases and Plasmas, Brownian Diffusion (Diffusiophoresis), Turbulent Diffusion, Neutron Diffusion, Photon Diffusion (Radiative Diffusion).

April 11 (a) George Misium, EECS Dept, UCB, "The Complete Design
T Procedure for Symmetric RF Discharges"
T,E,S (b) Dr. Ian Morey, EECS Dept., UCB, "Short Review of the Workshop on ICRF-Edge Plasmas" held at Boulder CO, March 30-April 1.

April 18 (a) Prof. Reiner Stenzel, Physics Dept. UCLA, "Instabilities and
E Emission Lines of Sheath Plasma Resonances"
T (b) Prof. S. Self, Stanford, continuation of April 4 talk on boundary conditions.

April 25 (a) Arthur Sato, Physics Dept. UCB, "The Plasma Resonance Probe,
T,E with works of Harp, Kino, Pavcovich et al."
T,S (b) Richard Procassini, NE Dept., UCB, The One-Dimensional Magnetized Sheath", based on works by R. Chodura et al.

May 2 (a) Dr. Steve Savas, Applied Materials Corp, Santa Clara," TEM Surface Waves in 13.56MHz Plasma Process Discharges"
(b) Dr. Kim Theilhaber, UCB, now LLNL, "Theory and Simulations of the Two-Dimensional Magnetized Sheath, Including Kelvin-Helmholtz Instability and Growth to large "Steady" Vortices"

May 5 (Special seminar, out of sequence, visitor)
Dr. Rod Boswell, Australian National Univ., Canberra, "Plasma Processing with Inductively Coupled RF Plasma Discharges"

May 9 (a) Dr. Ian Morey, EECE Dept., UCB, "Particle Simulations of Parallel Plate RF Discharges" (works done at ANU, Canberra)
(b) Prof. C.K. Birdsall, EECS Dept., UCB "Review of Sheath and Boundary Layer Models, as presented in this seminar.
(c) End of the seminar party, somewhere nearby.

B. PLASMA SHEATHS AND OTHER BOUNDARIES Seminar

CHARACTERIZATIONS

We will be reviewing a few dozen works this semester, with a main objective of finding which sheath and boundary layer models work and which do not, as learned from theory, experiment and simulation. We will be wise to follow some kind of **characterization** in these reviews, looking for the details of the models. Hence, some such will now be suggested, and hopefully followed, making comparisons easier.

General characterizations:

number of dimensions: 1, 2, or 3
time independent (dc)
time dependent (ac)
not magnetized
magnetized (usually applied dc B_0)
driven (battery, RF, other)

Particle characterizations (electrons, ions, neutrals)

sources, sinks; at walls, in the volume
after creation, how treated:
electrons: particles, fluid, Boltzmann, cut-off Maxwellian, other
ions: similarly, or cold, warm, beam, other
neutrals: temperature, pressure, excited states, other
equations of motion include terms for (list)
equations of continuity include terms for (list)

Wall characterizations (metal, dielectric, vacuum, another plasma, etc.)

floating, no net currrent
connected to external circuits and dc/ac sources by wires, antennas, waveguides, optical fibers
absorbing, reflecting, emitting
sputtering, etching, depositing

Characterization of results (from theory, experiments, simulations)

potentials, fields, densities, pressures, currents, with critical values for stable/unstable regions, changes of state, other
wave propagation, linear, nonlinear
oscillations, fluctuations, special features (like virtual cathodes)
fluxes of particles, energy, heat to walls
scalings obtained (e.g., particle flux to wall goes as $1/B_0$), or in lengths, time, amplitudes.

These characterizations are meant as guidelines, sort of a recipe for making notes while reading the articles and preparing transparencies for presentations. They are intended to make the task efficient as well as suggesting a common mode for the presentations. Lastly, they give us a ready way of comparing the models made and tested by various authors and aid in pinpointing room for improvement. Please give them a try.

Our colleague S. Kuhn in Innsbruck (plus visits here) published the chart below in one of his works, showing a particular set of characterizations. You might find making such charts useful also.

TABLE II. Comparison of some representative contributions to the kinetic theory of linear longitudinal plasma oscillations in one-dimensional geometry.

CRITERION	REFERENCE —	Buts 1971	Derfler ... 1966	Dobrowolny ... 1969	Ender ... 1979	Evans ... 1970	Evans 1970	Evans ... 1971	Faulkner ... 1969	Iizuka ... 1970	Kuznetsov ... 1980	Landau (§1) 1946	Landau (§2) 1946	Montgomery ... 1962	Pierce 1944	Pierce 1948	Rosenbluth ... 1963	present
		33	31	13	24	19	18	20	32	34	25	1a	1b	30	28	29	23	P
1. MODEL	cold-fluid	+	+			+		+	+	+					+	+		(+)
	kinetic			+	+		+				+	+	+	+			+	+
2. NUMBER OF BOUNDARIES	zero											+						(+)
	one					+	+	+					+					(+)
	two	+	+	+	+				+	+	+			+	+	+	+	+
3. PLASMA EQUILIBRIUM	uniform					+	+	+	+	+		+	+	+	+	+		(+)
	unif./thin sh.			+														+
	non-uniform	+	+		+						+						+	
4. SPATIAL DEPENDENCE	inf.-syst. modes								+	+		+			+	+		(+)
	Fourier series			+										+				(+)
	Laplace transf.					+	+	+										+
	integral eq.	+	+		+						+		+				+	
5. TIME DEPENDENCE	$\exp(\alpha t)$	+	+	+	+				+	+	+		+		+	+	+	(+)
	δ pulse	+																(+)
	Laplace transf.					+	+	+				+		+				+
6. PARTICLE BOUNDARY CONDITIONS	const. emission	+	+	+	+	+			+	+	+	+	+		+	+	+	(+)
	plain absorption	+	+	+	+	+	?	?	+	+	+				+	+	+	(+)
	spec. reflection			+									+	+				(+)
	other																	+
7. A.C. EXTERNAL CIRCUIT	short	+			+				+	+	+	+		+	+	+	+	(+)
	open			?		+	?	?					+					(+)
	passive		+															(+)
	active	+	+															+

C. University of California, Berkeley

14 January 1988

This is a list of references begun by C. K. Birdsall*for use in our Spring 1988 interdisciplinary plasma seminar,

PLASMA SHEATHS AND OTHER BOUNDARY LAYER DYNAMICS

The references are given by year, author, and journal, with some connotations, like theory, experiment and or simuation (T,E,S), and other comments. We will draw on the list already begun by our Innsbruck colleague, S. Kuhn, and by our Plasma Theory and Simulation Group members, especially Lou Ann Schwager.

We are interested in **unusual plasma regions**, with large potential energies, $e\phi > KT$, with appreciable charge imbalance, $n_i \neq n_e$, with relatively steep gradients in densities $n_{0,e,i}$, in potentials and fields ϕ, E, and in temperature T. We expect wall charges, wall absorption, emission and reflection, along with coupling to external circuits, sources. The plasma regions may have elastic and ineleastic collisions (scattering, exciting, ionizing, radiating, etc.) as well as generation of particles by all means.

Such regions show up in sheaths, at probes, in double layers, at the edges of freely expanding plasmas, both magnetized and not. All laboratory and space plasmas have some kind of boundaries someplace, most of which will influence the interior behavior to some extent. These regions are labelled unusual only in the sense that they do not obey the contraints which are commonly put on plasma models in order to make the models tractable. Indeed, it may be that what we are labelling unusual may be more common. It may well be that the final understanding of plasmas may come from understanding what is really going on at the plasma boundaries. It is certainly true that what we know about plasmas is obtained by making contact through plasma boundaries.

*Note: The list attached is not the "final list" that I plan to prepare during the seminar. It is a copy of works collected by Dr. Siegbert Kuhn, our associate in Innsbruck. As you will note, there are only 5 from the 1920's through the 1950's; there are 50 in the 1960's, 34 in the 1970's and, so far, 86 in the 1980's. Please bring additions to my attention.

Kuhn's list plus Birdsall's sheath collection are now typed in REFER on our local computer. We will publish this list soon.

SECTION V: JOURNAL ARTICLES, REPORTS, TALKS, VISITS

Journal Articles

Niels F. Otani and Bruce I. Cohen, "Effect of Large-Amplitude Perpendicularly Propagating Radio Frequency Waves on the Interchange Instability," Phys. Fluids, 31, pp. 158-176, January 1988.

Niels F. Otani, "Application of nonlinear dynamical invariants in a single electromagnetic wave to the study of the Alfven-ion-cyclotron instability," *Phys. Fluids*, 31 June 1988, pp. 1456-1464.

Reports

K. Theilhaber, "Vortex Formation and Particle Transport in a Cross-Field Plasma Sheath," University of California, Berkeley, Memorandum No. UCB/ERL M88/21, March 20, 1988.

Lou Ann Schwager and C. K. Birdsall, "Collector and Source Sheaths of a Finite Ion Temperature Plasma," University of California, Berkeley, Memorandum No. UCB/ERL M8/23, April 13, 1988.

Lou Ann Schwager, "Effects of Secondary Electron Emission on the Collector and Source Sheaths of a Finite Ion Temperature Plasma," University of California, Berkeley, Memorandum No. UCB/ERL M88/24, April 13, 1988.

Lou Ann Schwager, "Effects of Ion Reflection on the Collector and Source Sheaths of a Finite Ion Temperature Plasma," University of California, Berkeley, Memorandum No. UCB/ERL M88/25, April 13, 1988.

Poster Papers

ICRF/Edge Physics Workshop, March 30-April 1, 1988, Boulder, Colorado.

W. S. Lawson and C. K. Birdsall, "Undriven Plasma Wall Interaction Simulations, Showing Turbulence with and without an Initial Vacuum Gap."

W. S. Lawson and C. K Birdsall, "Antenna Driven Plasma Wall Interaction Simulation, Showing Local Turbulence and About 3 Times Larger Flux to the Wall."

IEEE Conference on Plasma Sciences, June 6-8, 1988, Seattle, Washington. (Abstracts follow)

W. S. Lawson, M. A. Lieberman, and C. K. Birdsall, "Electron Dynamics of RF Driven Parallel Plane Reactor."

A. Friedman, S. L. Ray, C. K. Birdsall, and S. E. Parker, "Multi-Scale Particle-in-Cell Plasma Simulation: Timestep Control Criteria and some Tests."

Talks

C.K. Birdsall, "Plasma-Sheath-Surface Dynamics via Particle Simulations," June 9-10, 1988, at NSF Workshop on New Directions in Plasma Engineering.

Electron Dynamics of RF Driven Parallel Plane Reactor

by William S. Lawson, M. A. Lieberman, and Charles K. Birdsall
Electronics Research Lab, Univ. of Calif., Berkeley CA 94720

Self-consistent many-particle simulations have been used to test some aspects of the low pressure model for the parallel plate reactor proposed by Godyak [1]. This model assumes warm mobile electrons and uniform immobile ions. One element of this model is the heating of electrons by the moving sheaths at the ends of the device. This heating was seen by Kushner in simulations in which the electric field was imposed [2], but not in our simulations in which the electric field was computed self-consistently. Our finding is that the self-consistent field in the one-dimensional geometry is just that required to prevent this heating.

Our simulations were initiated with a Maxwellian velocity distribution of electrons uniform in density ($\omega_p = 1$) between the driving plates. The system length was $L = 100$. Runs were made with $v_t = 2.0, 1.0, 0.5, 0.0$, so that the system lengths were $50\lambda_D$ and up. The model included no collisional or ionization effects. The plates were driven by a sinusoidal current at a frequency of $\omega = 0.1\omega_p$ with an amplitude chosen to produce an average sheath width of $.1L$. During the initial transient (the first half cycle), about 24% of the electrons are lost to the walls (20% is predicted). Most of the particle loss is from the ends of the the distribution, but a small component comes from the loss of fast particles from inside the distribution.

After the initial transient the electron dynamics are quite striking:

1) The electrons oscillate incompressibly in synchrony with the sheath edges; this motion precludes Fermi acceleration heating of the electrons by the sheath fields — see the $v - x$ plot below;

2) The electron velocity distribution becomes a cut-off Maxwellian, and only one or two electrons are lost each time the distribution comes close to the plates;

3) The average sheath thickness is roughly the $0.1L$ predicted;

4) The dominant frequency of the voltage between the driving plates is the driving frequency ω, but there is a persistent small oscillation at ω_p — see $\phi(t)$ plot below for $t = 600$ to 800;

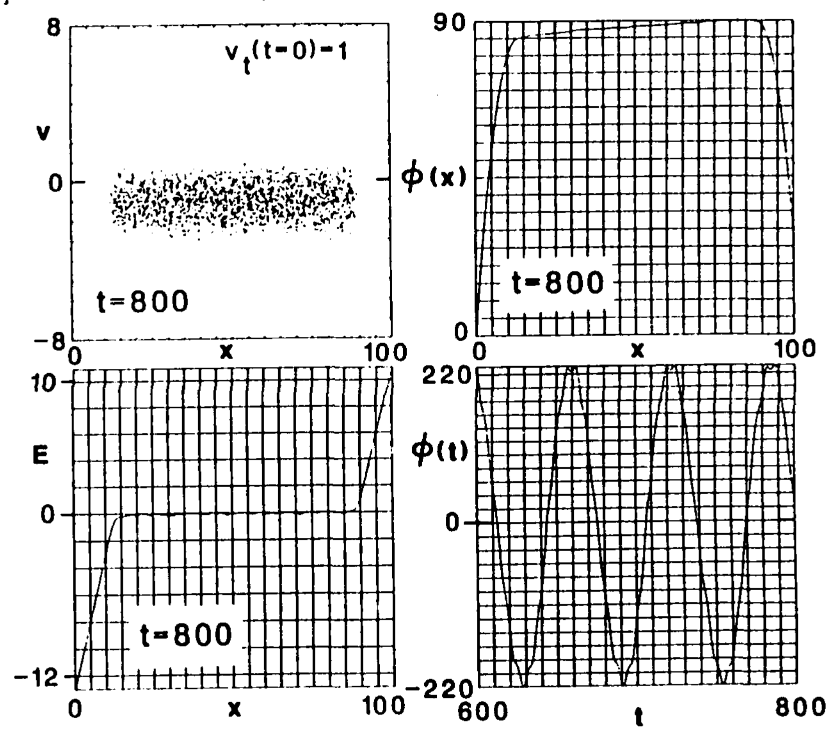
5) The peak voltage is at the predicted amplitude when $v_t = 0$, but increases slowly with the thermal velocity so that at $v_t = 2$ it is 30% higher than predicted.

Models with non-uniform ion densities or non-unidimensional geometry may produce heating at sheath edges, but any theory predicting the magnitude of this heating must predict the absence of heating in the one-dimensional model. Such a theory has been produced by Lieberman [3].

[1] V. A. Godyak, *Sov. Phys.-Tech. Phys.* **16**, 1073 (1972)

[2] M. J. Kushner, *IEEE Trans. Plasma Sci.*, **PS-14**, 188 (1986)

[3] M. A. Lieberman, submitted to *IEEE Trans. Plasma Sci.*, (1988)



MULTI-SCALE PARTICLE-IN-CELL PLASMA SIMULATION: TIMESTEP CONTROL CRITERIA AND SOME TESTS*

Alex Friedman and Scott L. Ray, *LLNL*
C. K. (Ned) Birdsall and Scott E. Parker, *UC Berkeley*

Implicit particle simulations [1–4] can be used to model macroscopic systems. However, at large timestep the spatial and temporal resolution is limited. Thus, a small timestep is necessary whenever one must resolve a fine space or time scale anywhere within the domain.

We are developing a new implicit simulation technique [5,6] which relaxes these restrictions and is suitable for strongly inhomogeneous problems involving a wide range of space and time scales. The plasma in any part of phase space is advanced on its own natural scale. The majority of the particles are not processed during any given step; for some problems this gain in economy may be two orders of magnitude.

Particles are advanced with a timestep $j\Delta t$, where $j = 2^m$; j is large for particles in regions where the field is smooth. As particles move about in phase space, their timestep sizes change. Here we describe two approaches to timestep control.

The position error after a step of size Δt which assumed a constant acceleration over its duration is proportional to $\dot{a}\Delta t^3$; this error should be a small fraction of some characteristic length L. The simplest timestep control employs a finite-difference approximation to $\dot{a}$, and sets L to of order the distance moved by the particle in a step. Denoting the previous step size by Δt and the desired new "reference" step size by τ, we have: $(|a^n - a^{n-1}|/\Delta t)\tau^3 = \beta^2 v\tau$, where v is a "reference velocity" and β an input accuracy criterion. If $\Delta t > \gamma\tau$ we halve the step size, while if $\Delta t < \tau/\gamma$ we double it. Here γ is a constant $\geq \sqrt{2}$; use of a value ≈ 1.5 introduces a useful "hysteresis" into the step size calculation. For v we use either $(|v^n| + |v^{n-1}|)/2$ or $(|v^n + v^{n-1}|)/2$.

We have tested this scheme in a simple code which uses both leapfrog and *d1* [4] movers to advance particles in a power-law potential such as $\phi = x^4$. We find that, while the scheme is usable, it suffers from a delay in the step-size adjustment due to the use of a^{n-1} in deriving τ.

Another scheme is based upon the assumption that $\dot{a} \sim v \cdot \nabla a \equiv \beta^2 \omega_{tr}^2$. The simplest implementation is just $\tau = \beta/\omega_{tr}$; with this we indeed observe better performance. In our testbed code MIST we enforce a stricter timestep control on fast particles by way of the variant: $\tau = (\beta/\omega_{tr})\{v_{th}/(|v^n| + |v^{n-1}| + \epsilon v_{th})\}^\alpha$. Here v_{th} is the thermal velocity (ideally computed locally), ϵ is a constant of order unity, and $\alpha = 1/2$ is a natural choice. For ω_{tr} we use the maximum of $|\nabla E|$ on the mesh over a neighborhood of order the maximum single-step excursion.

We are testing MIST on problems such as plasma expansion into vacuum. Some calculations exploring timestep control will be described.

1. R. J. Mason, *J. Comput. Phys.* **41** (1981), 233.
2. J. Denavit, *J. Comput. Phys.* **42** (1981), 337.
3. A. Friedman, A. B. Langdon, and B. I. Cohen, *Comm. Plasma Phys. Controlled Fusion* **6** (1981), 225.
4. A. B. Langdon and D. C. Barnes, in "Multiple Time Scales" (J. U. Brackbill and B. I. Cohen, Eds.), Academic Press, Orlando, 1985.
5. A. Friedman, Proc. US-Japan Workshop on Advanced Plasma Modeling, Nagoya, Japan, March 23-26, 1987.
6. A. Friedman, S. L. Ray, C. K. Birdsall, and S. E. Parker, Proc. 12th. Conf. on Numerical Simulation of Plasmas, Paper CW-6, San Francisco, Sept. 20-24, 1987.

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